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Quasilinear theory of Cherenkov-drift instability

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We examine linear and quasiliner stages of Cherenkov-drift instability developed in the relativistic magnetized electron-positron plasma penetrated by ultrarelativistic beam of electrons (or positrons). The plasma flow is streaming along the slightly curved magnetic field lines. In this case, the curvature drift of beam particles plays a decisive role in the development of the instability. A quasilinear relaxation of Cherenkov-drift instability leads to diffusion of resonant particles in momenta space. The expressions for diffusion coefficients of Cherenkov-drift instability are obtained. The numerical estimations are carried out for the parameters of relativistic magnetized plasma of pulsar magnetospheres providing a test of validity of the approximations used in our approach.

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I. INTRODUCTION

Cherenkov-drift instability was suggested by Kazbegi, Machabeli and Melikidze [1–4] as a possible mechanism for the generation of pulsar radio emission and later it was approved in Ref. [5]. In those works the linear theory of Cherenkov-drift instability was developed. It was shown, that due to Cherenkov-drift instability in pulsar magnetospheres the orthogonally polarized plasma waves are excited. These waves can escape from the magnetosphere and reach an observer as a pulsar radio emission. The necessary condition for the development of Cherenkov-drift instability (as for an usual Cherenkov instability) is a presence of a beam of particles in the relativistic magnetized pair plasma (consisted of relativistic electrons e^- and positrons e^+).

Generally, Cherenkov type instabilities develop due to a resonant interaction between waves and particles of a beam. The resonance occurs, when the electric field vector $\mathbf{E}$ and the wave vector $\mathbf{k}$ of generated waves have got components along direction of the beam velocity $\mathbf{v}$ ($\mathbf{E}\cdot\mathbf{v}\neq 0$ and $\mathbf{k}\cdot\mathbf{v}\neq 0$). As follows, the transverse waves ($\mathbf{E}\perp\mathbf{k}$), propagating along the external magnetic field ($\mathbf{B}_0\|\mathbf{k}$), cannot be generated by the usual Cherenkov instability (because it develops on the beam particles moving along the external straight magnetic field lines: $\mathbf{v}\|\mathbf{B}_0$ and $\mathbf{E}\cdot\mathbf{v}=0$).

Cherenkov-drift instability develops when the beam particles move along slightly curved magnetic field (SCMF) lines and, hence, drift across the plane where the curved lines lie. The drift motion of the beam particles provokes the generation of a purely *transverse* as well as the *longitudinal-transverse* waves.

Generally there are two most important effects caused by the particle relativistic motion along the SCMF line: curvature drift and curvature radiation. The drift velocity is directed across the plane of the SCMF lines and is given by the following expression:

$$u_d=\frac{\gamma v_{\|}^2}{\omega_B R_B}. \tag{1}$$

Here $\omega_B=qB/mc$ is the cyclotron frequency of a particle of charge q and mass m; R_B is the curvature radius of the magnetic field line; γ is a Lorentz factor of a particle; c is the speed of light and $v_{\|}$ is the component of $\mathbf{v}$ along the magnetic field line. If $\gamma\gg 1$, the value of drift velocity u_d can be significant. Let us note that electrons and ions are drifting in opposite directions.

A single particle, moving along the curved magnetic field line, radiates so called curvature radiation which can be easily described as a synchrotron radiation in an effective magnetic field (see, e.g., Ref. [6]). In 1975 Blandford [7] investigated the curvature radiation of plasma flowing along the SCMF lines. The problem was studied in the limit of infinite magnetic field $B_0\to\infty$, and it was shown that there is no radiation at all: the waves, radiated by each particle, are absorbed by another one. This result was confirmed later in papers [2,8,9]. Although in the paper by Asseo, Pellat, and Sol [10], the possibility of the waves excitation was shown, but the curved field-aligned beam was assumed to be finite in extend and immersed in the external plasma or vacuum having a sharp boundary at the edge. If the plasma flow has zero width, the instability is reduced to that of Goldreich-Keeley [11].

The necessity of taking into account the drift motion for the analysis of the reabsorption of curvature radiation was emphasized in Refs. [9,12,13]. Actually, Chugunov and Shaposhnikov [13] were the first who demonstrated that the particle beam drifting across the curved magnetic field lines

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is capable of amplifying electromagnetic radiation (the effect of maser curvature emission). The authors applied the result for pulsar magnetospheres and suggested the maser curvature emission as a mechanism of a pulsar radio emission. The problem was solved for *monoenergetic* beam of electrons, which means that all particles "felt" the waves of one and the same phase. It seems reasonable to suppose, that such setting of the problem has only theoretical interest: existence of strongly monoenergetic beam in plasma of pulsar magnetospheres is far from reality. Moreover, the effective wave-particle interaction (i.e., negative reabsorption) should be problematic, if the beam particles' distribution function has not a smooth slope in its shape. The monoenergetic beam results in a very narrow resonance width, $(\omega-\mathbf{kv})\rightarrow 0$. Thus, the growth rate of the excited waves will be insignificant [9]. The authors consider electromagnetic waves having the maximum frequency equal to that of single particle curvature radiation, $\omega_c \sim c\gamma/R_B$. However, in order to resolve the spectra of the waves, generated by the instabilities, the plasma collective processes should be taken into account. The mechanism of maser curvature radiation is based on the mechanism of a single particle curvature radiation and does not consider the effects of wave-particle interaction. That is why the viability of the application of maser curvature radiation to pulsars remains uncertain.

The development of Cherenkov-type instability, including the curvature drift velocity of the beam was studied and the growth rate was calculated in Refs. [1,2,4,14–18]. These results were confirmed later after a thorough investigation of the problem in Refs. [5,19]. The instability was called a Cherenkov-drift instability. Let us mention that the presence of the curved magnetic field lines is the necessary condition for both the Cherenkov-drift radiation in plasma and the single particle curvature radiation in vacuum. However, the Cherenkov-drift radiation cannot be interpreted as a "plasma curvature radiation," analogous to the single particle curvature radiation: a single particle radiates even for the infinite intensity of magnetic field, when $B_0 \rightarrow \infty$ and the drift velocity $(u_d \propto 1/B_0)$ vanishes in this limit. In contrast, the Cherenkov-drift radiation is not generated if $u_d \approx 0$. Moreover, the single particle radiates the vacuum wave, while the Cherenkov-drift instability generates the proper modes of the medium (i.e., relativistic electron-positron plasma). This particular point was not considered by Blandford [7] and Melrose [8]. Polarization of these waves strongly differs from that of vacuum waves.

A brief examination of the linear theory of Cherenkov-drift instability is discussed in Sec. II. In Sec. III the quasilinear equations for the Cherenkov-drift instability are obtained. Validity of the approximations, using for the analysis of the quasilinear equations, is examined for the plasma parameters of typical pulsar magnetospheres. In Sec. IV coefficients describing the diffusion of particles in momentum space are evaluated. Alteration of plasma distribution function is studied. The results are summarized in Sec. V.

II. THE LINEAR THEORY

Properties of magnetized relativistic electron-positron plasma penetrated by a beam of ultrarelativistic particles

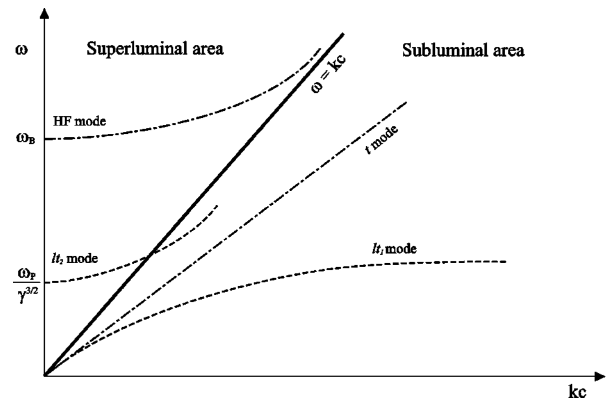

FIG. 1. The cylindrical frame of reference (x, r, φ) and a local Cartesian frame of reference (x, r, y). The x axis is directed up from the plane of the figure (the positive direction is chosen toward the drift of electrons). $\mathbf{B}_0$ is the external magnetic field; R_B is the radius of curvature of a magnetic field line.

were carefully investigated in a series of works [20–23] and are still the subject of concern [24,25]. The strong magnetic field constrains plasma motion in parallel direction to magnetic field lines. The electrons and positrons lose their perpendicular energy rapidly due to synchrotron radiation and are assumed to be in their lowest Landau orbitals. Hence their motion is essentially one dimensional along the field lines. In our particular case, we consider that the magnetic field exhibits a weak inhomogeneity

$$\frac{1}{B_0}\frac{\partial B_0}{\partial R} \ll \frac{1}{L}, \tag{2}$$

implying that the characteristic length scale (L) of the system is considerably less then the characteristic dimensions of the magnetic field inhomogeneity. To put it in another way, the plasma flow is streaming along the SCMF lines the local torsion of which can be neglected:

$$R_B \gg L. \tag{3}$$

In doing that the *local* spatial inhomogeneities of the magnetic field and the plasma flow are of no importance below.

The dielectric permittivity tensor, $\epsilon_{ij}(\omega,\mathbf{k})$, for such a medium was derived in Ref. [3]. The problem was considered in cylindrical coordinates, (x, r, φ). The x axis is directed perpendicularly to the plane of the magnetic field curvature (the positive direction of x axis is chosen toward the drift of electrons) while r and φ are the radial and the azimuthal coordinates, respectively (see Fig. 1). Hence, φ component of a vector is parallel to the magnetic field line.

The electron-positron plasma differs from the electron-ion plasma by lack of gyrotropy. Consequently, spectra of waves propagating in the e^-e^+ plasma is simpler than that in the electron-ion plasma. It consists of only four types of waves that correspond to four branches on the diagram $\omega(k)$, where ω is the wave frequency (see Fig. 2).

One of them is high-frequency transverse electromagnetic wave totally located in superluminal area. Its phase velocity

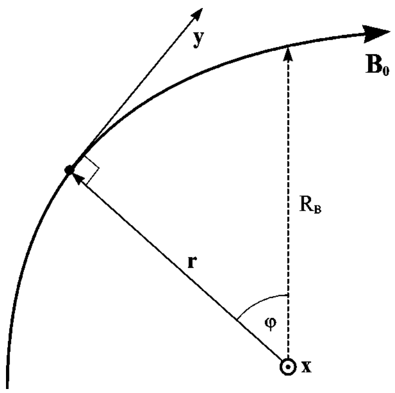

FIG. 2. Dispersion curves of the waves in the relativistic magnetized electron-positron plasma. The solid line corresponds to $\omega = kc$. It divides the plane (ω, kc) into superluminal and subluminal areas. The high-frequency branch of t waves (in the superluminal area) is defined as high-frequency (HF) mode. Low-frequency and high-frequency modes of lt waves are defined as lt_1 and lt_2 modes, respectively.

$v_{ph} = \omega/k$ exceeds the speed of light c, hence, it is not of interest in our discussions below.

The second branch represents the dispersion of a purely transversed linearly polarized electromagnetic wave. The dispersion of this wave is described by the following equation:

$$\omega_0^t = kc(1-\delta), \tag{4}$$

(where $\delta = \omega_p^2/4\omega_B^2\gamma_p^3$; $\omega_p^2 = 4\pi q^2 n_p/m$ is plasma Langmuir frequency). We call this dispersion curve as t mode. It is totally located in subluminal area, and therefore could be generated by particles of a beam. Its electric field vector $\mathbf{E}^t$ is perpendicular to the plane of wave vector and external magnetic field $(\mathbf{k}, \mathbf{B}_0)$.

The remaining two dispersion curves on the $\omega(k)$ diagram describe the longitudinal-transverse waves propagating in relativistic e^-e^+ plasma. One of them is almost superluminal. This wave is purely longitudinal if it propagates strictly along the magnetic field line $(\mathbf{k}\|\mathbf{E}\|\mathbf{B}_0)$, and, in this case called Langmuir wave associated with longitudinal oscillations of the charge density. If an angle $\vartheta \approx k_\perp/k_\|$ between $\mathbf{k}$ and $\mathbf{B}_0$ increases ($k_\perp$ and $k_\|$ are the components of wave vector across and along the magnetic field line, respectively), the component of $\mathbf{E}$ starts to grow across $\mathbf{k}$: Langmuir wave transforms to the longitudinal-transverse wave denoted as lt_2 mode in Fig. 2. If the angle ϑ is small enough, $\vartheta \leqslant \vartheta_0 \sim \sqrt{\delta}$, lt_2 mode is almost longitudinal and crosses $\omega = kc$ line. In this case, lt_2 mode can be excited if the Cherenkov resonance condition $\omega = k_\| v_\|$ fulfills. However, for the resonant particles of primary beam, the growth rate of the instability is very small [26]. The wave leaves from the interaction area so quickly that no time is left for significant amplification of the wave. In the case of oblique propagation, $\vartheta > \vartheta_0$, lt_2 mode is totally superluminal. Therefore, lt_2 mode could not be generated at all by particles of a beam.

Another longitudinal-transverse wave is denoted as lt_1 mode in Fig. 2. Dispersion equation of this mode is the following:

$$\omega_0^{lt} = k_\| c\left(1 - \delta - \frac{k_\perp^2 c^2}{16\omega_p^2\gamma_p}\right). \tag{5}$$

This mode, like t wave, is located totally in subluminal area, and can easily be generated by plasma particles. Its electric field vector $\mathbf{E}^{lt}$ is located in $(\mathbf{k}, \mathbf{B}_0)$ plane. The lt_1 mode is vacuum wave if it propagates along the magnetic field lines $(\mathbf{k}\|\mathbf{B}_0)$. Its dispersion curve merges with t mode (see Fig. 2) and can be arbitrarily polarized. In the case of oblique propagation, electric field of lt_1 wave has the component $E_\|^{lt}$ along the external magnetic field, thereby involving plasma particles in longitudinal oscillations.

Generation of lt_1 mode, propagating perpendicular to the plane of SCMF lines, is connected with the drift motion of the particles. These waves are also known as *drift waves* [2,27,28].

It should be mentioned that, while describing the waves in relativistic e^-e^+ plasma, some authors sometimes use terminology which, in our opinion, appears to be misleading. For example, since the work by Arons and Barnard [22], the superluminal longitudinal-transverse wave (lt_2 mode) was called an ordinary (O) mode, the subluminal transverse wave (t mode)—an extraordinary (X) mode and the subluminal longitudinal-transverse wave (lt_1 mode)—an Alfvén mode. However "ordinary" and "extraordinary" are generally related to the waves propagating across the external magnetic field in the usual electron-ion plasma [29]. Moreover, t wave (the so called X mode) is the purely transverse wave and its analog does not exist in the electron-ion plasma. As for the Alfvén mode, in electron-ion plasma such a name is used for an almost linearly polarized, transverse electromagnetic wave with frequency $\omega \ll \omega_{B_i}$ (where ω_{B_i} is the cyclotron frequency of ions). In the case of $k \to \infty$ and $\omega \lesssim \omega_{B_e}$, the $\omega(k)$ curve splits into two branches describing dispersions of (a) right-hand polarized electron-cyclotron waves with frequency $\omega \approx \omega_{B_e}$ and (b) left-hand polarized ion-cyclotron waves with frequency $\omega \approx \omega_{B_i}$. Therefore, in relativistic e^-e^+ plasma, we prefer to call the dispersion curves t, lt_2, and lt_1 modes, respectively, hence avoiding a possible confusion with dispersion curves in electron-ion plasma.

In the papers by Kazbegi *et al.*, [2–4], it was shown that t and lt waves could be generated by particles of the beam when the following resonance condition is satisfied:

$$\omega - k_\varphi v_\varphi - k_x u_d = 0. \tag{6}$$

For transverse (4) and longitudinal-transverse (5) plasma waves the resonance conditions can be written as follows:

$$\left(2\delta - \frac{k_r^2}{k_\varphi^2}\right) = \left(\frac{u_d}{c} - \frac{k_x}{k_\varphi}\right)^2 \tag{7a}$$

and

$$\left(\frac{u_d}{c}-\frac{k_x}{k_\varphi}\right)^2=\left(2\delta+\frac{k_\perp^2c^2}{8\omega_p^2\gamma_p}+\frac{k_x^2}{k_\varphi^2}\right), \tag{7b}$$

respectively [27].

If the above conditions (7) are fulfilled, we can estimate the resonance value of emission beam width ($\vartheta\approx k_\perp/k_\parallel$) as

$$\vartheta_0\approx\frac{u_0}{c}, \tag{8}$$

where u_0 is the resonant value of u_d. The expressions for the resonant frequency and the growth rate of excited waves can be written as follows:

$$\omega_0=\frac{c\gamma_0\omega_b}{u_0\gamma_T^{3/2}}, \tag{9}$$

$$\Gamma_0=\frac{\pi}{2}\frac{\omega_b^2}{\omega_0}\frac{\gamma_0}{\gamma_T^2}A, \tag{10}$$

where $A=(k_r/k_\perp)^2$ for t-waves and $A=(k_x/k_\perp)^2$ for lt waves; γ_T and ω_b are thermal spread and Langmuir frequency of resonant particles of the beam, respectively. These expressions were obtained in papers [1,4,15]. As for the growth rate of the drift wave, it was obtained in paper [2],

$$\Gamma_d=\frac{\omega_b}{\omega_p}\left(\frac{3}{2}\frac{\gamma_p^3}{\gamma_0}\right)^{1/2}k_xu_0. \tag{11}$$

The reason for generation of t, lt, and drift waves is the presence of the beam in the relativistic pair plasma, although the waves cannot be excited without the drift motion of particles. Indeed, expressions (10) and (11) are equal to zero if $u_d=0$. All those waves are purely or almost transverse waves. For t and lt_1 waves the electric field vector is perpendicular to the external magnetic field [see Eq. 10]. As for the waves generated by the usual beam instability, both their electric field vector $\mathbf{E}$ and the wave vector $\mathbf{k}$ are directed along the external magnetic field $\mathbf{B}_0$.

In the following section we study the quasilinear equations which significantly differ from those of the usual beam-plasma instability.

III. QUASILINEAR EQUATIONS

To study the quasilinear theory of Cherenkov-drift instability, we use the collisionless kinetic equation of the following form:

$$\frac{\partial f}{\partial t}+\mathbf{v}\frac{\partial f}{\partial \mathbf{r}}+\frac{\partial}{\partial \mathbf{p}}\left[\frac{q}{mc}\left(\mathbf{E}+\frac{\mathbf{p}\times\mathbf{B}}{\gamma}\right)f\right]=0, \tag{12}$$

where $f\equiv f(\mathbf{r},\mathbf{p},t)$ is the distribution function of the particles.

According to the standard scheme, in order to obtain a set of quasilinear equations, distribution function as well as electric and magnetic field vectors have to be divided into the main and oscillating parts: $f(\mathbf{r},\mathbf{p},t)=f_0(\mathbf{p},\mu t)+f_1(\mathbf{r},\mathbf{p},t)$, $\mathbf{E}(\mathbf{r},t)=\mathbf{E}_1(\mathbf{r},t)$, and $\mathbf{B}(\mathbf{r},t)=\mathbf{B}_0(\mu t)+\mathbf{B}_1(\mathbf{r},t)$. Then, averaging equation (12) over fast oscillations and assuming $\langle f_1\rangle=\langle\mathbf{B}_1\rangle=\langle\mathbf{E}_1\rangle=0$, $\langle f_0\rangle=f_0$, and $f_0\gg f_1$, the following equations can be obtained:

$$\frac{\partial f_0}{\partial \mu t}=-\left\langle\frac{q}{mc}\frac{\partial}{\partial\mathbf{p}}\left(\mathbf{E}_1+\frac{\mathbf{p}\times\mathbf{B}_1}{\gamma}\right)f_1\right\rangle\equiv QL, \tag{13a}$$

$$\frac{\partial f_1}{\partial t}+\frac{c\mathbf{p}}{\gamma}\frac{\partial f_1}{\partial\mathbf{r}}+\frac{q}{mc}\left(\frac{\mathbf{p}\times\mathbf{B}_0}{\gamma}\right)\frac{\partial f_1}{\partial\mathbf{p}}=-\frac{q}{mc}\left(\mathbf{E}_1+\frac{\mathbf{p}\times\mathbf{B}_1}{\gamma}\right)\frac{\partial f_0}{\partial\mathbf{p}}. \tag{13b}$$

Here $\langle\cdots\rangle$ denotes averaging over fast oscillations; $\partial/\partial\mu t$ is a slow time derivative ($\mu\ll 1$). The slow local spatial variations of f_0 and B_0 are neglected within the length scale of the system L [see Eqs. (2) and (3)]. Equation (13a) describes the back reaction of generated waves upon the nonperturbed distribution function f_0. In order to calculate the quasilinear term QL it is enough to substitute the solution of Eq. (13b) into Eq. (13a). The solution of Eq. (13b) in the Fourier presentation,

$$f_1(\mathbf{r},\mathbf{p},t)=\frac{1}{(2\pi)^3}\int f_{\mathbf{k}}(\mathbf{p})\exp(i\mathbf{k}\cdot\mathbf{r}-i\omega_{\mathbf{k}}t)d\mathbf{k}, \tag{14}$$

reads

$$f_{\mathbf{k}}(\mathbf{p})=-\frac{q}{mc}\int_{-\infty}^{t}dt'\exp[i\mathbf{k}\cdot\boldsymbol{\rho}-i\omega_{\mathbf{k}}\tau]\left[\mathbf{E}_{\mathbf{k}}\left(1-\frac{\mathbf{p}'}{\gamma}\cdot\frac{\mathbf{k}c}{\omega_{\mathbf{k}}}\right)+\frac{(\mathbf{E}_{\mathbf{k}}\cdot\mathbf{p}')}{\gamma}\frac{\mathbf{k}c}{\omega_{\mathbf{k}}}\right]\frac{\partial f_0}{\partial\mathbf{p}'}, \tag{15}$$

where $\omega_{\mathbf{k}}\equiv\omega(\mathbf{k})+i\Gamma_{\mathbf{k}}$; $\boldsymbol{\rho}=\mathbf{r}'-\mathbf{r}$, and $\tau=t'-t$. Then, perturbed electric field vector $\mathbf{E}_1$ is substituted for perturbed magnetic field $\mathbf{B}_1$ using Maxwell equation $\mathbf{B}_{\mathbf{k}}=(c/\omega_{\mathbf{k}})(\mathbf{k}\times\mathbf{E}_{\mathbf{k}})$ for the Fourier transforms:

$$\mathbf{E}_1(\mathbf{r},t)=\frac{1}{(2\pi)^3}\int \mathbf{E}_{\mathbf{k}}\exp(i\mathbf{k}\cdot\mathbf{r}-i\omega_{\mathbf{k}}t)d\mathbf{k};$$

$$\mathbf{B}_1(\mathbf{r},t)=\frac{1}{(2\pi)^3}\int \mathbf{B}_{\mathbf{k}}\exp(i\mathbf{k}\cdot\mathbf{r}-i\omega_{\mathbf{k}}t)d\mathbf{k}. \tag{16}$$

In order to find $f_{\mathbf{k}}(\mathbf{p})$, we are using a standard method of integration along nonperturbed trajectories. Following the method, in Eq. (15) $(\mathbf{r}',\mathbf{p}')$ are phase coordinates of the particle (along the nonperturbed trajectory) at the instant of time t'; they are calculated from the relativistic equations of motion of a single particle:

$$\frac{d\mathbf{r}'}{dt'}=\frac{c}{\gamma}\mathbf{p}', \tag{17a}$$

$$\frac{d\mathbf{p}'}{dt'}=\frac{q}{mc}\left(\frac{\mathbf{p}'\times\mathbf{B}_0(\mathbf{r}',t')}{\gamma}\right). \tag{17b}$$

Using a cylindrical frame of reference, the nonperturbed magnetic field is modeled as $\mathbf{B}_0=\mathbf{B}_0(0,0,B_\varphi)$. However, as we mentioned above, local description allows to neglect torsion of the curve magnetic field lines [see Eq. (3)]. So we can switch to a local Cartesian frame of reference, where cylindrical φ coordinate is measured along y axis, parallel to a local magnetic field direction (see Fig. 1). This approach is valid to study the Cherenkov-drift instability, if the area of wave-particle interaction (that can be taken as a characteristic scale length of the system, L) is greater than the length of excited waves, $\lambda\sim c/\Gamma_0$:

$$\vartheta_0 R_B \gtrsim \frac{c}{\Gamma_0}. \tag{18}$$

Condition (18), as well as condition (3), are well satisfied for the parameters of plasma of typical pulsar magnetospheres. The electron-positron plasma of pulsar magnetospheres consists of mainly two components: a bulk plasma of particles with low Lorentz factors ($\gamma_p\sim 3-10$) and a beam of primary particles ejected from the stellar surface with Lorentz factors $\gamma_b\sim 10^6-10^7$. A number density of the beam is $n_b\sim n_{GJ}$, where n_{GJ} is the so called Goldreich-Julian density. A number density of a bulk plasma is n_p ($n_p/n_b\sim\Sigma$, where $\Sigma\sim 10^2-10^7$ is the Sturrock "multiplication factor" [30]). At Cherenkov-drift instability, developed in typical pulsar magnetospheres, where $R_B\approx 10^9$ cm, the opening angle of excited waves ϑ_0 reaches the values $\approx 0.06-0.1$ [see Eq. (7)] and $\Gamma_0\approx 6\times 10^2-10^3\ \mathrm{s}^{-1}$ [assuming $A\approx 1$ and $\gamma_T\approx 10^4$ in Eq. (10)]. The instability develops at the distances $R\approx 0.5R_{LC}-0.8R_{LC}$ from the pulsar, where $R_{LC}=cP/2\pi$ is the radius of light cylinder and P is the period of pulsar rotation. For typical pulsars ($P=1$ sec) $R_{LC}\approx 5\times 10^9$ cm. The value of dipolar magnetic field is estimated as $B=B_s(R_s/R)^3$ and equal to $B\approx 20-60$ G in the area where the instability develops (here $B_s\sim 10^{12}$ G is the value of the magnetic field at the surface of a pulsar with a radius of the order of $R_s\sim 10^6$ cm). In such a magnetic field the cyclotron frequency of a relativistic electron is $\omega_B\approx 3\times 10^8-10^9\ \mathrm{s}^{-1}$ and, consequently, the drift velocity of the resonant particles of a beam (with $\gamma_0\sim 10^6$) obtains the values of the order of $u_0/c\sim 10^{-2}-10^{-1}$. Substituting the resonant values of parameters (8)–(10), condition (18) can be written in the form useful for estimations in magnetized relativistic plasma,

$$\frac{R_B}{c}\frac{\omega_B^2}{\omega_b}\frac{\gamma_T^{1/2}}{\gamma_0^2}<1. \tag{19}$$

The set of equations of motion (17) is solved using the approximation of locally straight field lines, although it is taken into account that the particles undergo the drift with characteristic velocity u_d [see Eq. (1)]. Using the geometry shown in Fig. 1, the relativistic equation of motion (17b) is rewritten as the following set of equations:

$$\frac{dp_x'}{dt'}-\tilde{\omega}_B p_r'=0,$$

$$\frac{dp_r'}{dt'}-p_\varphi'\frac{d\varphi'}{dt'}+\tilde{\omega}_B p_x'=0,$$

$$\frac{dp_\varphi'}{dt'}+p_r'\frac{d\varphi'}{dt'}=0, \tag{20}$$

where $\tilde{\omega}_B=\omega_B/\gamma$. Ignoring the terms proportional to the small parameter $r_L/R_B\approx v_\perp u_d/c^2\ll 1$ [where $v_\perp^2=(v_x-u_d)^2+v_r^2$ and $r_L=v_\perp/\tilde{\omega}_B$ is the radius of Larmor circle] and assuming that $p_\varphi^2\gg(p_r^2+p_x^2)$, we can obtain the solutions of set of Eqs. (20) as was done in Ref. [17]:

$$p_x'=p_d+p_r\sin(\tilde{\omega}_B\tau)+(p_x-p_d)\cos(\tilde{\omega}_B\tau),$$

$$p_r'=p_r\cos(\tilde{\omega}_B\tau)-(p_x-p_d)\sin(\tilde{\omega}_B\tau),$$

$$p_\varphi'=p_\varphi. \tag{21}$$

Here the components of dimensionless momentum (p_x, p_r, p_φ) are the values of (p_x', p_r', p_φ') at the instant of time $t'=t$, and $p_d=(u_d/c)\gamma$. We have the following integrals of motion: $\gamma, p_x'-(\tilde{\omega}_B r'/c)\gamma, r'p_\varphi'$. Let us notice that the particle distribution function f_0 should only depend on the integrals of motion.

It is evident that the solutions of equation of motion (21) differ from those in the homogeneous magnetic field (with straight magnetic field lines) only by the drift component p_d. On the basis of straight magnetic field approximation, we can locally accept cylindrical coordinates in momentum space ($p_\parallel, p_\perp, \theta$) as well. Subscripts "$\parallel$" and "$\perp$" denote parallel and perpendicular directions to the magnetic field $\mathbf{B}_0$, respectively,

$$p_\perp\cos\theta=p_x-p_d,$$

$$p_\perp\sin\theta=p_r.$$

Therefore, Eqs. (21) are reduced to the following form:

$$p_x'=p_d+p_\perp\cos(\theta-\tilde{\omega}_B\tau),$$

$$p_r'=p_\perp\sin(\theta-\tilde{\omega}_B\tau),$$

$$p_\varphi'=p_\parallel. \tag{22}$$

Substituting Eqs. (22) into Eq. (17a), we obtain the following expressions for $\boldsymbol{\rho}=\mathbf{r}'-\mathbf{r}$:

$$\rho_x=\frac{cp_d}{\gamma}\tau-\frac{cp_\perp}{\gamma}\frac{1}{\tilde{\omega}_B}[\sin(\theta-\tilde{\omega}_B\tau)-\sin\theta],$$

$$\rho_r=\frac{cp_\perp}{\gamma}\frac{1}{\tilde{\omega}_B}[\cos(\theta-\tilde{\omega}_B\tau)-\cos\theta],$$

$$\rho_\varphi=\frac{cp_\parallel}{\gamma}\tau. \tag{23}$$

On the basis of the same assumptions, we can accept translational symmetry of the system as well and substitute expressions (22) and (23) for the components of $\boldsymbol{\rho}$ and $\mathbf{p}'$ in Eq. (15) (see also Ref. [5]). Hence, we finally obtain the Fourier transform of oscillating distribution function $f_{\mathbf{k}}(\mathbf{p})$:

$$f_{\mathbf{k}}(\mathbf{p})=-\left(\frac{q}{mc}\right)\frac{i\exp[ib\sin(\theta-\phi)]}{\Delta\omega_{\mathbf{k}}}\left[\frac{\partial f_0}{\partial p_\parallel}\left(E_\parallel(\mathbf{k})+\frac{k_\parallel c}{\omega_{\mathbf{k}}}\frac{p_d}{\gamma}E_x(\mathbf{k})\right)+\frac{\partial f_0}{\partial p_\perp}\left(E_\parallel(\mathbf{k})\frac{2p_d\cos\theta}{\gamma}+\frac{k_\parallel c}{\omega_{\mathbf{k}}}\frac{p_d}{\gamma}E_x(\mathbf{k})\frac{2p_d\cos\theta-p_\perp}{\gamma}\right)\right], \quad (24)$$

where $\Delta\omega_{\mathbf{k}}\equiv\omega_{\mathbf{k}}-k_\parallel v_\parallel-k_x u_d$. Calculating expression (24) from Eq. (15), we used the following presentation of the exponential function:

$$\exp[i\mathbf{k}\cdot\boldsymbol{\rho}-i\omega_{\mathbf{k}}\tau]=\exp[ib\sin(\theta-\phi)]\sum_{n=-\infty}^{\infty}\mathcal{E}_n J_n(b). \quad (25)$$

Here $\mathcal{E}_n\equiv\exp[in(\phi-\theta)-i\tau(\Delta\omega_{\mathbf{k}}-n\tilde{\omega}_B)]$; $J_n(b)(n=0;\pm1;\pm2\ldots)$ is the Bessel function of integer order [31];

$$b=k_\perp r_L \quad (26)$$

and ϕ is defined as follows:

$$k_x=k_\perp\cos\phi,$$

$$k_r=k_\perp\sin\phi. \quad (27)$$

In the derivation of Eq. (24), we have taken into account that $b\ll1$ and kept only the first order terms in the expansions of $J_n(b)$. Then we consider that $n=0$ in Eq. (25), since this approximation leaves only the terms describing the contribution of Cherenkov-drift resonance. Let us mention that $(\partial f_0/\partial\theta)=0$ since the distribution function possesses an axial symmetry. Therefore, the corresponding terms do not contribute into Eq. (24).

IV. QUASILINEAR DIFFUSION

The next step is to study the alteration of slowly varying part of distribution function $f_0(\mathbf{p},\mu t)$ due to the development of Cherenkov-drift instability. Generally, an alteration is described by diffusion coefficients involved in the quasilinear term. The coefficients show the rate of particle diffusion in momentum space along, as well as across, the magnetic field. Substituting Eq. (24) into Eq. (13a) and using Maxwell equation for Fourier transforms, we obtain QL term in the following form:

$$QL\equiv-\frac{q}{mc}\left\langle\frac{\partial}{\partial\mathbf{p}}\left(\mathbf{E}_1+\frac{\mathbf{p}\times\mathbf{B}_1}{\gamma}\right)f_1\right\rangle=\frac{\partial}{\partial p_\parallel}\left(D_{\parallel\parallel}\frac{\partial f_0}{\partial p_\parallel}\right)+\frac{\partial}{\partial p_\parallel}\left(p_\perp D_{\parallel\perp}\frac{\partial f_0}{\partial p_\perp}\right)+\frac{1}{p_\perp}\frac{\partial}{\partial p_\perp}\left(p_\perp^2 D_{\perp\parallel}\frac{\partial f_0}{\partial p_\parallel}\right)+\frac{1}{p_\perp}\frac{\partial}{\partial p_\perp}\left(p_\perp D_{\perp\perp}\frac{\partial f_0}{\partial p_\perp}\right). \quad (28)$$

Here $D_{\parallel\parallel}$, $D_{\parallel\perp}$, $D_{\perp\parallel}$, and $D_{\perp\perp}$ are diffusion coefficients. Below we calculate some particular expressions for diffusion coefficients corresponding to t and lt waves in the case of Cherenkov-drift instability. Let us notice, that Eq. (28) has rather general meaning and can be used for other type of instabilities as well. However, the diffusion coefficients will be different for different instabilities.

It is convenient to consider quasilinear development of t and lt waves separately. If we assume that these waves are purely electromagnetic $(\mathbf{E}\perp\mathbf{k})$, then the following relations between components of electric field and wave vector can be written as

$$E_\parallel=0,\quad E_x k_x=-E_r k_r, \quad (29)$$

for t waves and

$$E_x k_r=E_r k_x,\qquad E_\parallel k_\parallel=-E_x k_x, \quad (30)$$

for lt waves (if $k_x\gg k_r$). Using Eqs. (29) and (30) we can write the diffusion coefficients for t and lt waves as:

$$D_{\parallel\parallel}=\int_{-\infty}^{\infty}U_\parallel^2\,\mathcal{I}(\mathbf{k})\,d\mathbf{k}; \quad (31a)$$

$$D_{\perp\perp}=\int_{-\infty}^{\infty}U_\perp^2\,\mathcal{I}(\mathbf{k})\,d\mathbf{k}; \quad (31b)$$

$$D_{\parallel\perp}=D_{\perp\parallel}=0, \quad (31c)$$

where we use the following definitions:

$$U_\parallel^2=\left(\frac{k_\parallel c}{\omega_{\mathbf{k}}}\frac{p_d}{\gamma}\right)^2,\quad U_\perp^2=\frac{k_\parallel c}{\omega_{\mathbf{k}}}\left(\frac{p_d}{\gamma}\right)^2\left(1-\frac{k_\parallel c}{\omega_{\mathbf{k}}}\right), \quad (32a)$$

for transverse t waves;

$$U_\parallel^2=\left(\frac{k_\parallel c}{\omega_{\mathbf{k}}}\frac{p_d}{\gamma}+\frac{k_x}{k_\parallel}\right)^2,$$

$$U_\perp^2=\left(\frac{k_\parallel c}{\omega_{\mathbf{k}}}\frac{p_d}{\gamma}+\frac{k_x}{k_\parallel}\right)\left(\frac{p_d}{\gamma}+\frac{k_\parallel c}{\omega_{\mathbf{k}}}\frac{k_x}{k_\parallel}\right)\left(1-\frac{k_\parallel c}{\omega_{\mathbf{k}}}\right), \quad (32b)$$

for longitudinal-transverse lt waves;

$$\mathcal{I}(\mathbf{k})\equiv i\frac{1}{\Delta\omega_{\mathbf{k}}}\left(\frac{q}{mc}\right)^2\frac{E_x(-\mathbf{k})E_x(\mathbf{k})}{V(2\pi)^3}. \quad (33)$$

Note that equations for diffusion coefficients (31) along with Eqs. (32) and (33) are obtained after averaging the expression within brackets $\langle \cdots \rangle$ [see Eq. (13a)] over the angle θ. This procedure nullifies the diffusion coefficients $D_{\parallel\perp}$ and $D_{\perp\parallel}$ taking into account that the corresponding terms

$$\frac{\partial}{\partial p_\parallel}\left(p_\perp D_{\parallel\perp}\frac{\partial f_0}{\partial p_\perp}\right) \quad \text{and} \quad \frac{1}{p_\perp}\frac{\partial}{\partial p_\perp}\left(p_\perp^2 D_{\perp\parallel}\frac{\partial f_0}{\partial p_\parallel}\right)$$

are smaller than the terms

$$\frac{\partial}{\partial p_\parallel}\left(D_{\parallel\parallel}\frac{\partial f_0}{\partial p_\parallel}\right) \quad \text{and} \quad \frac{1}{p_\perp}\frac{\partial}{\partial p_\perp}\left(p_\perp D_{\perp\perp}\frac{\partial f_0}{\partial p_\parallel}\right)$$

by a factor of $J_1(b) \approx b \ll 1$. In Eq. (33), $E_x-(\mathbf{k})$ identifies the complex conjugate to the x component of electric field vector $\mathbf{E_k}$, and $V=\int_{-\infty}^{\infty} d\mathbf{r}$.

It is worth noting, that drift velocity (1) depends on Lorentz factors of the particles, γ. Hence, thermal spread in the Lorentz factors of resonant particles ($\gamma_T = |\gamma - \gamma_0|$) results in the scatter of drift velocities, increasing the resonant width of instability, $\Delta\omega = \omega - k_\parallel v_\parallel - k_x u_d$. It allows to consider the kinetic approximation of Cherenkov-drift instability. However, the resonant width of usual Cherenkov instability is smaller than the corresponding width of Cherenkov-drift instability. Therefore, in the case of the presence of narrow relativistic beam (with low value of γ_T), the kinetic approximation for usual Cherenkov instability is not valid. The growth rate of the instability, $\Gamma_\mathbf{k}$, is small for hydrodynamic approximation as well. Therefore usual Cherenkov instability, as opposed to Cherenkov-drift instability, cannot develop in relativistic magnetized pair plasma [23,26].

Particle diffusion in momentum space appears in both parallel and perpendicular directions with respect to the magnetic field B_0. The diffusion causes an alteration of particle distribution function until the quasilinear relaxation of instability is saturated ($\partial f_0/\partial\mu t = 0$, where f_0 is the distribution function of resonant particles). In order to investigate the quasilinear relaxation of f_0, it is worth to rewrite Eq. (13a) in the following form:

$$\frac{\partial f_0}{\partial \mu t} = \frac{\partial}{\partial p_\parallel}\left(D_{\parallel\parallel}\frac{\partial f_0}{\partial p_\parallel}\right) - \frac{1}{p_\perp}\frac{\partial}{\partial p_\perp}p_\perp\left(D_{\perp\perp}\frac{\partial f_0}{\partial p_\perp}\right). \quad (34)$$

It is easy to find out that the diffusion coefficients, in some sense, define the rate of alteration of f_0. To estimate the values of $D_{\parallel\parallel}$ and $D_{\perp\perp}$ for the plasma parameters of typical pulsar magnetospheres, we rewrite Eqs. (31) in the following simplified form:

$$D_{\parallel,\perp} \simeq 8\pi U_{\parallel,\perp}^2 \left(\frac{q}{mc}\right)^2 \frac{W}{\Gamma}, \quad (35)$$

where $U_\parallel^2 \approx (u_d/c)^2 \sim 10^{-4}$ and $U_\perp^2 \approx (u_d/c)^4 \sim 10^{-8}$; the energy of excited waves is about $W/W_p \sim 10^{-2}$, where $W_p \equiv mc^2 n_p \gamma_p$; $n_p \approx 3\times 10^8$ cm^{-3}; $\gamma_p \approx 3$. Finally, we obtain that $D_{\parallel\parallel} \sim 10^{10}$ s^{-1} and $D_{\perp\perp} \sim 10^6$ s^{-1}.

Assuming that $k_\parallel c \sim \omega_\mathbf{k}$, $k_x/k_\parallel \sim p_d/\gamma$ and $(1 - k_\parallel c/\omega_\mathbf{k}) \sim (p_d/\gamma)^2$ in Eqs. (32), we can rewrite Eq. (34) as follows:

$$\frac{\partial f_0}{\partial \mu t} = \left[\frac{\partial}{\partial p_\parallel}\left(\frac{p_d}{\gamma}\right)^2 \frac{\partial f_0}{\partial p_\parallel} + \frac{1}{p_\perp}\frac{\partial}{\partial p_\perp}p_\perp\left(\frac{p_d}{\gamma}\right)^4 \frac{\partial f_0}{\partial p_\perp}\right] \times \int_{-\infty}^{\infty} g\mathcal{I}(\mathbf{k})\,d\mathbf{k}, \quad (36)$$

where $g=1$ for t waves and $g=4$ for lt waves. The stationary state ($\partial f_0/\partial\mu t = 0$) depends on relative importance between right-hand terms in Eq. (36). The ratio between the terms describing parallel and perpendicular diffusions is of the order of $\sim (p_\perp/p_d)^2$. Therefore, we can consider the following two cases.

(a) $p_\perp \gg p_d$. In this case the first right-hand term in Eq. (36) significantly exceeds the second term. Hence, the quasilinear relaxation is saturated by plateau formation on parallel distribution function of resonant particles ($\partial f_0/\partial p_\parallel = 0$).

(b) $p_\perp \ll p_d$. In this case the second right-hand term in Eq. (36) appears significant. The quasilinear relaxation causes energy transfer from parallel flow to the perpendicular motion of the particles, hence increasing $p_\perp$. The relaxation will be saturated when $p_\perp \sim p_d$. This is the case when both right-hand terms in Eq. (36) are of the same order and, consequently, cancel each other (here, we are bearing in mind that in the resonance region the beam particle distribution function has a negative slope of $\partial f_0/\partial p_\perp < 0$).

These results are natural: back reaction of excited waves over resonant particles should suppress the reason of wave excitation. In the case of Cherenkov-drift instability, it causes the plateau formation on the distribution function of parallel momenta (similar to the case of quasilinear relaxation of usual Cherenkov instability) and energy transfer from parallel flow of particles to their motion across the magnetic field. The later process, inhibits an anisotropy in momentum space. It is similar to the quasilinear relaxation of cyclotron instability [32]. [The reason for development of cyclotron instability—anisotropy in momentum space ($p_\perp \ll p_\parallel$)—is suppressed by particle diffusion over perpendicular momenta. As a result, the energy of parallel motion of the beam particles is transferring into the perpendicular energy until $p_\perp \sim p_\parallel$.]

V. CONCLUSION

In summary, we conclude that the development of Cherenkov-drift instability causes the diffusion of beam particles (confined to one-dimensional motion along the strong, slightly curved magnetic field lines) both *across* and along the magnetic field lines. We take into account the curvature drift motion and study the quasilinear theory of Cherenkov-drift instability that reveals the perpendicular diffusion of the resonant particles in momenta space. The expressions for diffusion coefficients are obtained and their values are estimated for the plasma parameters of typical pulsar magnetospheres. The numerical value of coefficients provide the rate of alteration of distribution function, f_0 [see Eq. [(36)]. It is shown that initially one-dimensional ($p_\perp \to 0$) distribution of beam particles is unstable relative to Cherenkov-drift instability. The quasilinear relaxation inhibits an anisotropy of f_0 increasing the transverse momenta, $p_\perp$. Finally, the relaxation is saturated since $p_\perp \sim p_d$.

This scenario works if the other factors which can balance the quasilinear diffusion are not taken into account. Such factors could be, on one hand, the radiation reaction force (acting on synchrotron emitting particle, spiraling in strong magnetic field) and, on the other hand, the force arising due to particle motion in weekly inhomogeneous field. We plan to include these factors into consideration in future works.

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